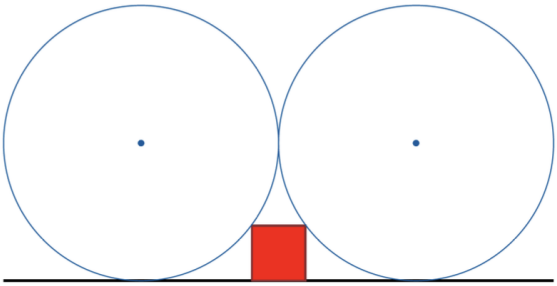
NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 4

Fall 2024

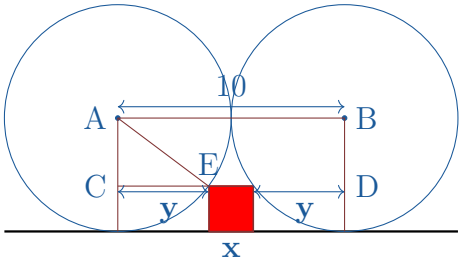
Problem 4

Find the area of the red square placed between the two circles of radius 5 meters in the diagram:



Justify your answer.

Solution. We first make the following construction



and set $\mathbf{x}$ as the side-length of the red square and $\mathbf{y}$ as the length of CE. From the picture it is clear that the length

$$\mathbf{y} + \mathbf{x} + \mathbf{y} = \mathbf{x} + 2\mathbf{y} = 10. \tag{1}$$

Now we notice that in the right-angled triangle ΔACE, $|\mathrm{AC}| = 5 - \mathbf{x}$ and $|\mathrm{AE}| = 5$. Thus by the Pythagorus theorem we have

$$(\mathbf{x} - 5)^2 + \mathbf{y}^2 = 5^2. \tag{2}$$

Substituting the value of $\mathbf{y}$ from (1), we get

$$\begin{aligned} (\mathbf{x} - 5)^2 + \left(\frac{10 - \mathbf{x}}{2}\right)^2 &= 25 \\ \Rightarrow \mathbf{x}^2 - 12\mathbf{x} + 20 &= 0 \\ \Rightarrow (\mathbf{x} - 2)(\mathbf{x} - 10) &= 0 \\ \Rightarrow \mathbf{x} &= 2 \text{ or } 10. \end{aligned}$$

Since $\mathbf{x} = 10$ is not possible, the side-length of the red square must equal 2 meters.